ID Number: *Key* Test 2 8:27 8:49 — 22

Calculator Allowed

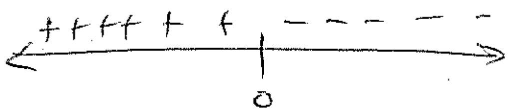
1. (~~10~~ ~~8~~ 7 points) You are given that $m(x) = \dfrac{-x^2}{5(x^2+10)}$, $m'(x) = \dfrac{-4x}{(x^2+10)^2}$, $m''(x) = \dfrac{12(x^2-4)}{(x^2+10)^3}$. Use calculus to answer the following questions:

(a) Does the graph of m have a local maximum or minimum? If so, what are its coordinates? State whether it is a maximum or a minimum.

Set $m'(x) = 0$ since denom is always ≥ 0

$-4x = 0 \Rightarrow x = 0$

$\boxed{\text{Yes, has a local } \underline{\text{max}} \text{ at } (0,0)}$



local *__max__*

(b) On what interval(s) is m increasing?

m is increasing on $(-\infty, 0)$.

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2. (~~10~~ points) You are given that $m(x) = \dfrac{-x^2}{5(x^2+10)}$, $m'(x) = \dfrac{-4x}{(x^2+10)^2}$, $m''(x) = \dfrac{12(x^2-4)}{(x^2+10)^3}$. Use calculus to answer the following questions:

(a) On what interval(s) (if any) is the graph of m concave downward?

Set $m''(x) = 0$ since denom $\neq 0$

$12(x^2-4) = 0$

$\Rightarrow 12(x-2)(x+2) = 0$

$x = 2$, $x = -2$

$\boxed{\text{Concave down on } (-2, 2)}$

+++ ——— ——— +++

−2 2

(b) On what interval(s) (if any) is the graph of m concave upward?

concave up on

$(-\infty, -2)$ and $(2, \infty)$

(c) Find the (x, y)-coordinates of each point of inflection for m. Answers *must* be written as ordered pairs.

points of inflection at $x = \pm 2$

$m(2) = \dfrac{-4}{5(14)} = \dfrac{-2}{35}$ $m(-2) = \dfrac{-2}{35}$ also

$\boxed{\left(2, \frac{-2}{35}\right) \quad \left(-2, \frac{-2}{35}\right)}$

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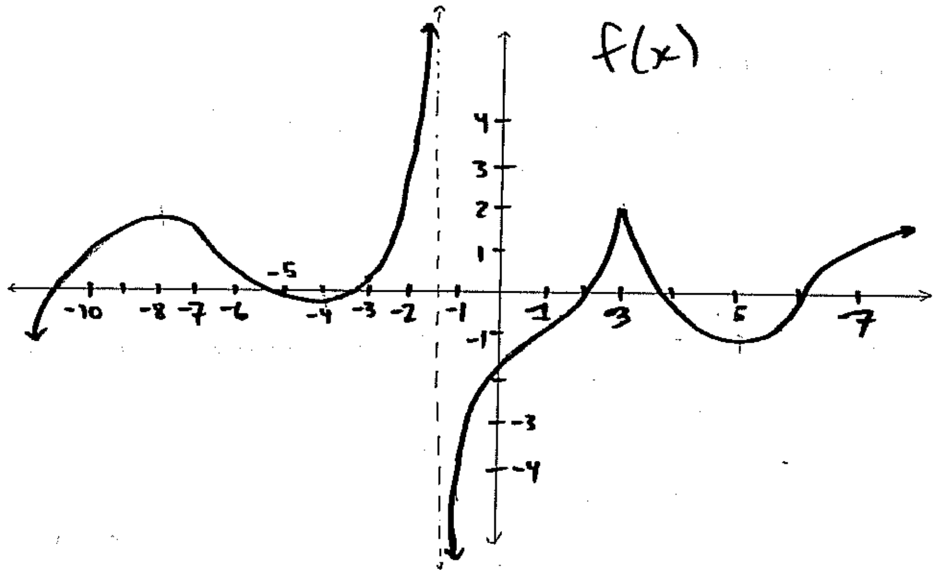
3. (~~6~~ points) Let $f(x)$ be represented by the following graph:



(a) On what interval(s) is $f(x)$ increasing? Decreasing?

Increasing : $(-\infty,-8)$, $(-4,-1.5)$, $(-1.5,3)$, $(5,\infty)$

Dec: $(-8,-4)$, $(3,5)$

(b) On what interval(s) is $f(x)$ concave downward?

$(-\infty,-6.5)$, $(-1.5,1)$, $(6,\infty)$

(c) Find the coordinates of all local maximums and minimums.

local max: $(-8,2)$, $(3,2)$

local min: $(-4,-0.5)$, $(5,-1)$

(d) Find the coordinates of all inflection points.

$(-6.5,1)$, $(1,-1)$, $(6,0)$

(e) At what ~~point(s)~~ x-values in the domain of f does $f'(x)$ not exist?

at ~~x=-1.52 and~~ $x=3$

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4. (~~10~~ points) The daily cost of producing x portable DVD players is approximated by the function

$$C(x) = 0.0001x^3 - 0.08x^2 + 40x + 5000.$$

(a) Find the marginal cost function.

(b) Find the marginal cost when $x = 200$. What meaning does it have?

(c) Find the marginal cost at $x = 300$ and $x = 400$. What does this tell you about how many DVD players you should produce?

(d) Find the average cost function $\bar{C}(x) = \dfrac{C(x)}{x}$, then calculate $\bar{C}'(x)$.

(e) Calculate $\bar{C}'(200)$. Does its value make sense with the previous information?

(a) $C'(x) = 0.0003x^2 - 0.16x + 40$

(b) $C'(200) = 20$. Means that the 201^{st} DVD player will cost $\approx \$20$ to manufacture.

(c) $C'(300) = \$14$

$C'(400) = \$24$

You should probably produce around 300 DVD players because that it when marginal cost is cheapest

(d) $\bar{C}(x) = 0.0001x^2 - 0.08x + 40 + \dfrac{5000}{x}$

$\bar{C}'(x) = 0.0002x - 0.08 - \dfrac{5000}{x^2}$

(e) $\bar{C}'(200) = -0.165$

Does make sense because the cost per next item is going down, so the average cost must be decreasing, which is what we see.

5. (8 points) The supply equation for radios is given by

$$p = s(x) = 0.3\sqrt{x} + 10$$

where x is the quantity supplied and p is the unit price in dollars. Use differentials to approximate the change in price when the quantity supplied increases from 10,000 to 10,500 units.

$dy = f'(s)\,dx$

$s'(x) = \frac{0.3}{2} x^{-1/2}$

$s'(10{,}000) = \frac{0.3}{2 \cdot 100} = 0.0015$

$\Rightarrow dy = (0.0015)(500)$

$dy = 0.75$

$dx = 10{,}500 - 10{,}000 = 500$

The prize ~~[illegible]~~ goes up about \$0.75

No calculators on this part

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6. (~~10~~ points) Find $f'(x)$ if

$$f(x) = \frac{(3x+2)^4}{x^2+1}$$

low dhi − hi dlo

You do not need to simplify your answer.

$$\boxed{\frac{(x^2+1)(4(3x+2)^3 \cdot 3) - (3x+2)^4 \cdot 2x}{(x^2+1)^2}}$$

7. (~~10~~ 12 points) Find the second derivative of the function $f(x) = (x^3 - 7)^8$. Simplify your answer by factoring the final expression.

$$f'(x) = 8(x^3-7)^7 \cdot 3x^2$$

$$= 24x^2(x^3-7)^7$$

$$f''(x) = 24\left[2x(x^3-7)^7 + x^2 \cdot 7(x^3-7)^6 \cdot 3x^2\right]$$

$$= 24(x^3-7)^6 x\left[2(x^3-7) + 21x^3\right]$$

$$= 24(x^3-7)^6 x\left[2x^3-14 + 21x^3\right]$$

$$= \boxed{24x(x^3-7)^6(23x^3-14)}$$

8. (6 points) Find $f'(x)$ if

$$f(x) = (4x^5 + 7)(x^2 + 7x + 10)$$

Simplify your answer.

$$f'(x) = 20x^4(x^2+7x+10) + (2x+7)(4x^5+7)$$

$$= 20x^6 + 140x^5 + 200x^4 + 10x^6 + 14x + 28x^5 + 49$$

$$= \boxed{30x^6 + 168x^5 + 200x^4 + 14x + 49}$$

Extra Credit(2 points) In what order would you apply the product, quotient, and/or chain rules for the derivative of the function

$$f(x) = \left[(2x+7)\left(\frac{(x^3-4)^3}{5x}\right)\right]^8$$

You do <u>not</u> need to actually calculate the derivative.

Chain, then product for inside the brackets

when doing product need to do quotient,

and inside quotient must do another chain!

$\boxed{\text{chain, product, quotient, chain}}$